Faulty Hypotheses AND HEDGE FUNDS

This paper, which won the 2005 AIMA Canada Research Award, gets under the hood of hedge fund returns to dispel myths about relative risk.

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There have been a number of extremely bad moments in the hedge fund sector, but at the top of the list is the fall of 1998 when Long-Term Capital Management lost 92% of its investors' capital due to a liquidity crisis. Such incidents have damaged the reputation of the entire industry and have prompted questions about the prudence of hedge fund investing more generally. Unfortunately, such moments are not restricted to hedge funds: for example, the shares of both Nortel and Enron experienced larger losses (measured from their high points) than did LTCM. In contrast, however, these events are not typically considered to have affected the suitability of the entire equity asset class.

Still, many researchers claim that portfolios of hedge funds expose investors to risks that can be surprising.[1] This warning stems from the higher kurtosis and more negative skew that is usually reported for hedge fund indices compared to traditional equity indices. According to most statistics textbooks, higher kurtosis means a distribution has extreme outliers, and negative skew implies these outliers occur on the downside. Based on these measures, the hedge fund sector would appear to be guilty as charged.

Where are the surprises?

We dispute this conclusion in a recent working paper by comparing the monthly returns from inception to December 2004 for the CSFB/Tremont and HFRI hedge fund indices with returns on the S&P and Nasdaq indices. We also look at the HFRI fund of funds index, since it is a good proxy for the actual experience of diversified hedge fund investors.[2]

Our argument is illustrated most clearly in graphical form. Figure 1 displays histograms for the three hedge fund indices while Figure 2 displays similar data for the two equity indices. Note the scales on each histogram are the same. It is immediately evident that the extreme returns for the hedge fund indices are indeed surprising, but not in the manner expected. The occurrence of extreme returns—negative or positive—has been much less frequent for the hedge fund indices than for the equity indices.

How can this be reconciled with the common perception that hedge funds expose investors to extreme risks? Some insight can be gained by closely examining the measures of risk upon which this view is usually based. The datasets in Table 1 confirm the typical

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positive kurtosis and negative skew reported by other researchers as evidence of extreme events. The low standard deviation for the hedge fund indices, however, would appear to say the opposite.

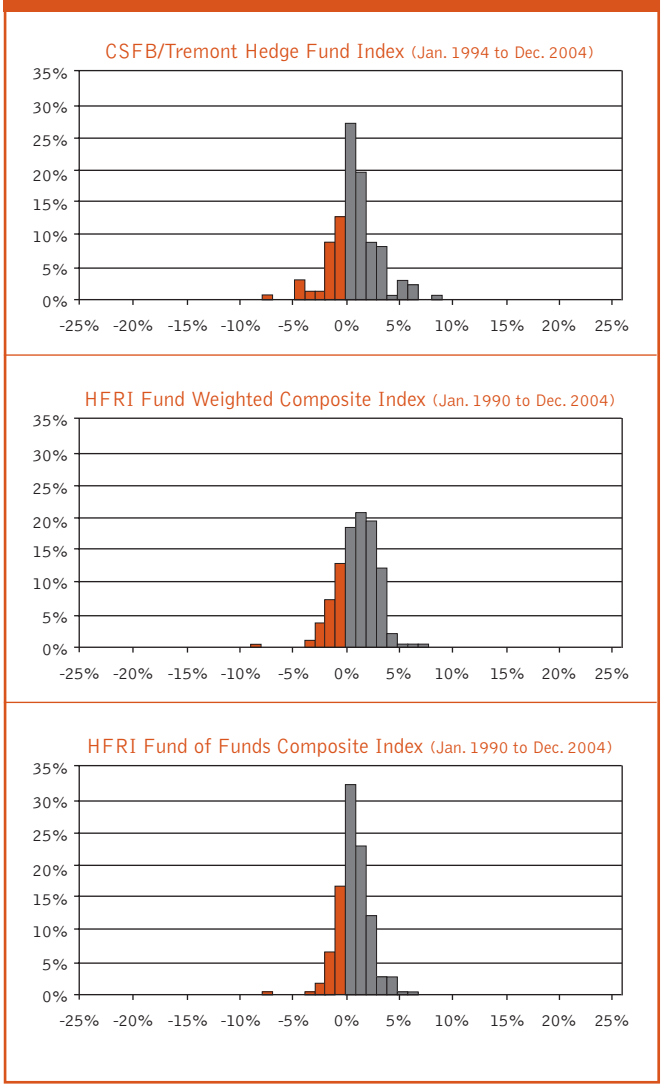
We resolve this confusion by identifying two difficulties with standard measures of skew and kurtosis. First, despite what is written in most textbooks, higher kurtosis cannot always be interpreted as indicating greater risk of extreme events. We provide numerical examples, based on a leading article published in 1945,[3] of distributions with high kurtosis yet lower area in the tails—and thus less risk of extreme events—when compared to a normal distribution with the same standard deviation. Second, the standard formulas for skew and kurtosis involve scaling the third and fourth statistical moments by the third and fourth powers of the standard deviation respectively.[4] When comparing distributions with standard deviations that are not the same, this begs the question of whether a high value of kurtosis is due to a large numerator (or moment) or a small denominator (or standard deviation). The histograms in Figure 1 imply it is the small standard deviation that is responsible.

Scale of substitution

When analyzing extreme event risk, we recommend the scaled measures of skew and kurtosis be replaced with the un-scaled third and fourth statistical moments. This approach has a firmly established foundation in investment theory because the third and fourth moments correspond directly to the third and fourth terms in a Taylor series expansion of an investor's expected utility function.[5] Applying this approach to the data yields surprising results as shown in Table 2. It is clear that the lower standard deviation in the denominator is driving the higher skew and kurtosis and leading to the incorrect inference that a portfolio of hedge funds exposes investors to extreme risks. The correct measures of extreme events—the third and fourth moments—are definitely surprising, but not in the way that is commonly reported.

FIGURE 1: HEDGE FUND INDICES

The scales are identical on both horizontal and vertical axes, with the horizontal showing monthly returns and vertical showing percentage of time in each bucket. The vertical axis is shown as percentage of returns so that different time periods can be compared. Note that the CSFB histogram is for a shorter timeframe than the other histograms.



We also consider the length and magnitude of maximum drawdown because of the common use of this measure in the industry. As discussed by Lamm (2003), the CSFB/Tremont index experienced a six standard deviation event during the fall of 1998 due to the "surprising moment" at LTCM mentioned above. This period represents the largest drawdown—13.8%—in the history of the index and took thirteen months to recover from. While this may seem severe, it is minor in comparison to the losses suffered by the stock markets since 2000. As Table 3 shows, the drawdowns for the hedge fund indices are smaller and the recovery times shorter; as of March 2005 neither equity index had yet recovered, and the recovery may still be years away, particularly for the Nasdaq. In short, Lamm was technically correct in noting this was a six-standard deviation event, but the focus should be on the low standard deviation and not on the "six."

What do investors need to know?

These findings have some surprising implications for investors. The first concerns the important question of what percentage allocation to hedge funds is appropriate. The magnitude of extreme returns on hedge fund indices has clearly been smaller than on equity indices over the time period considered. Each measure of risk reported in Table 2—standard deviation, third moment and fourth moment—is more favourable for the hedge fund indices than for the equity indices.[6]

As any diligent student of investment theory would quickly point out, the lower risk on hedge fund indices should be accompanied by lower returns. Tables 1 and 2 confirm this trade-off applies empirically for the CSFB/Tremont index and the HFRI fund of funds index, since their historical returns have been lower than the equity indices'. Remarkably, the returns for the HFRI fund weighted index are equal to or higher than the returns on the equity indices—despite the lower overall risk.

Long-standing models of portfolio choice, dating back to the seminal work of Harry Markowitz, rely on this risk-return tradeoff when determining appropriate allocations. Such models generally recommend that investments with the highest risk-return ratio be given a relatively large allocation, subject to how they correlate with other assets. Unfortunately, these models are not directly applicable to the process for determining appropriate hedge fund allocations; they work only if returns are normally distributed—and the higher kurtosis indicates hedge fund indices' returns are not.[7]

We resolve this difficulty by developing an investment decision rule which takes the higher statistical moments into account. This decision rule is based on an extension of the classic arbitrage argument where two portfolios are levered or de-levered so that they have equal risk; the one with the higher resulting return—which would be preferred by investors—can

FIGURE 2: EQUITY INDICES

The scales are identical on both horizontal and vertical axes, with the horizontal showing monthly returns and vertical showing percentage of time in each bucket. The vertical axis is shown as percentage of returns so that different time periods can be compared.

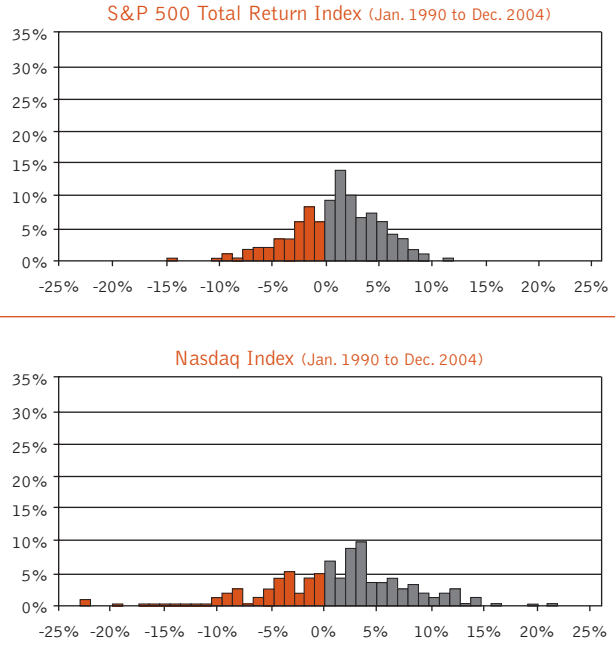


TABLE 1: STATISTICAL PROPERTIES OF EQUITY AND HEDGE FUND INDICES

All calculations are based on monthly returns. The data for the U.S. equity indices was downloaded from Yahoo and from Standard and Poor's. The data for the hedge fund indices was downloaded from the Tremont and HFRI websites.

Index	Mean (%)	Standard Deviation (%)	Skew	Kurtosis	Jarque-Bera Stat
Panel A — CSFB/Tremont (January 1994 to December 2004)					
U.S. Equity Indices					
S&P 500 Total Return	0.974	4.402	-0.611	3.486	9.52
Nasdaq	1.107	8.003	-0.373	3.664	5.48
CSFB/Tremont Hedge Fund Indices					
Hedge Fund Index	0.899	2.352	0.098	5.046	23.24
Panel B — HFRI (January 1990 to December 2004)					
U.S. Equity Indices					
S&P 500 Total Return	0.958	4.229	-0.470	3.644	9.74
Nasdaq	1.149	7.390	-0.390	3.889	10.50
HFRI Hedge Fund Indices					
Fund Weighted Composite	1.149	2.001	-0.626	5.898	74.76
Fund of Funds Composite	0.814	1.625	-0.257	7.305	140.99

then be identified. When returns are not normally distributed, levering or de-levering to equate only one aspect of risk—the fourth moment—may still provide guidance on which portfolio is preferable depending on the remaining moments of interest.[8]

The de-levered difference

Using this approach, we note that the CSFB/Tremont index and the HFRI fund of funds index are clearly preferable to the de-leveraged equity indices. This is because the returns are higher, the standard deviation is lower and the third moment is either positive or less negative. Thus an investor can make a clear choice in favour of these two hedge fund indices. The only exception is the HFRI fund weighted index, which has a third moment more negative than that of the Nasdaq. This index would likely still be preferred by most investors, however, because of a substantially higher return and lower standard deviation.

These results can be interpreted as evidence that large allocations to hedge funds are justified. We present additional evidence to support this conclusion even after adjusting for potential upward bias in hedge fund returns.[9] The favourable risk-return trade-off, combined with the lower overall risk for hedge fund indices as compared to equity indices, also implies a diversified portfolio of hedge funds is suitable for a broad range of risk tolerances—not just for "aggressive" investors as is typically thought.[10]

The second major implication of our work concerns the often misunderstood suitability of financial leverage. The results from our de-levering to match fourth moments provide support for the use of leverage on a diversified portfolio of hedge funds, as is the practice at some funds of hedge funds. This is evident by the significantly higher leverage ratios needed to equate the fourth moments for the hedge fund indices as compared to the equity indices.

Another implication is that investors need to exercise caution in using scaled measures of higher moments, such as the standard measure of skew and kurtosis. There is no theoretical basis for this scaling, and relying on these measures may lead to incorrect assessments of

TABLE 2: HIGHER MOMENTS OF EQUITY AND HEDGE FUND INDICES

The skew is calculated by dividing the third moment by the standard deviation^3, while the kurtosis is calculated by dividing the fourth moment by the standard deviation^4.

Index	Standard Deviation (%)	Skew	Kurtosis	Third Moment (%^{3})	Fourth Moment (%^{4})
Panel A — CSFB/Tremont (January 1994 to December 2004)					
U.S. Equity Indices					
S&P 500 Total Return	4.402	-0.611	3.486	-52.15	1,309.11
Nasdaq	8.003	-0.373	3.664	-191.13	15,028.66
CSFB/Tremont Hedge Fund Indices					
Hedge Fund Index	2.352	0.098	5.046	1.27	154.38
Panel B — HFRI (January 1990 to December 2004)					
U.S. Equity Indices					
S&P 500 Total Return	4.229	-0.470	3.644	-35.56	1,165.49
Nasdaq	7.390	-0.390	3.889	-157.52	11,596.31
HFRI Hedge Fund Indices					
Fund Weighted Composite	2.001	-0.626	5.898	-5.02	94.47
Fund of Funds Composite	1.625	-0.257	7.305	-1.10	50.92

the overall risk of a given investment.

Lastly, although we demonstrate that extreme returns on hedge funds are not problematic per se for investors, we also point out that applying standard mean variance portfolio tools to hedge funds may cause investors to overlook the risks associated with higher moments. This implication is drawn from statistics on various equity and hedge fund indices when leverage is applied in order to equate the variance: allocating to hedge funds on the basis of a risk budget that considers only variance does indeed expose an investor to fourth moments well in excess to those for the equity indices. This is because the trade-off between variance and the fourth moment is ignored in standard mean variance portfolio allocation tools when determining the appropriate level of overall risk. Thus it appears to be our tools, when applied to hedge funds, which expose investors to "hidden risk" and not the hedge funds themselves.[11]

Before concluding it is important to note a number of caveats. First, our decision rule does not consider combinations of the various equity and hedge fund indices. Second, it is important to note that "past performance is not indicative of future results," although the historical evidence in support of this argument appears to be clear. Third, the returns on individual hedge funds, which may be riskier than the returns on the hedge fund indices, have not been considered. As discussed earlier, however, investors are increasingly taking a portfolio approach in the hedge fund sector as they realize the benefits of diversification that can be obtained.

A moment's meditation

There have definitely been surprising moments in hedge fund investing. A number of individual hedge funds have experienced bad periods in the past and undoubtedly more difficulties will occur in the future. Analyzing hedge fund index data, however, shows that problems at any single fund have had a surprisingly small affect on returns for the hedge fund sector overall. In contrast, it is surprising how favourable for investors the statistical moments—

TABLE 3: PEAK-TO-TROUGH DRAWDOWNS AND RECOVERY TIMES

These calculations are based on month-end values—drawdowns may be larger when looking at daily data. Time to Recovery is the number of months the index took to move from the trough back to a level which matches or exceed its earlier peak. Still to be Recovered indicates indices have not yet recovered from their maximum drawdowns; it is calculated as the value at December 31, 2004 divided by its peak value minus 1.

Index	Peak to Drawdown (%)	Length of Drawdown (%)	Months to Recover	Still to be Recovered (%)
Panel A — CSFB/Tremont (January 1994 to December 2004)				
U.S. Equity Indices				
S&P 500 Total Return	-44.7	25	27	-14.5
Nasdaq	-75.0	31	27	-53.7
CSFB/Tremont Hedge Fund Indices				
Hedge Fund Index	-13.8	3	13	
Panel B — HFRI (January 1990 to December 2004)				
U.S. Equity Indices				
S&P 500 Total Return	-44.7	25	27	-14.5
Nasdaq	-75.0	31	27	-53.7
HFRI Hedge Fund Indices				
Fund Weighted Composite	-11.4	4	7	
Fund of Funds Composite	-13.1	6	12	

which properly measure the true nature of extreme returns—have actually been. ▮

Endnotes

1 See Brooks and Kat (2002), Lamm (2003) and Malkiel and Saha (2004), among others.

2 We also considered the various hedge fund sub-indices.

3 See "A Common Error Concerning Kurtosis" by Kaplansky (1945).

4 The denominators of the standard measures of skew and kurtosis are the third and fourth powers of the standard deviation, respectively. The numerators are the sum of the third and fourth powers of the deviations from the mean divided by the degrees of freedom, i.e., the third and fourth statistical moments. Note standard deviation is the square root of variance, which is the second statistical moment.

5 Scott and Horvath (1980) provide an example of this expansion.

6 Scott and Horvath (1980) show, under fairly general assumptions, that investors have a positive preference for return and third moments but dislike standard deviation and fourth moments.

7 Jarque-Bera statistics allow the hypothesis of normality to be rejected for all of the hedge fund indices.

8 Note the similarity to the early work of Rothschild and Stiglitz (1970) on stochastic dominance, which identifies very general conditions under which an investor would choose one risky asset over another without imposing distributional assumptions such as normality, or strong assumptions on preferences.

9 It is well known that hedge fund databases suffer from various biases. Malkiel and Saha (2004) construct a database that is relatively free from bias and report that hedge fund returns have been overstated by approximately 3.8% per year.

10 When determining suitability, there are other considerations such as liquidity that must also be taken into account.

11 Fung and Hsieh (1999) note hedge funds may appear to be very attractive from a mean variance perspective, but not when the skew and kurtosis are considered.

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